YHRD kinship formula

$$
\begin{aligned}
L &: \text{set of loci with typed results for ancestor and offspring} \\
s_l &: \text{allelic difference between ancestor and offspring at locus } l \\
\mu_{l,i} &: i\text{-step mutation rate at loci } l \\
\frac{\mu_{l,i}}{2} &: \text{probability of exactly one } i\text{-step mutation at locus } l \\
\left(\frac{\mu_{l,i}}{2}\right)^{s_l} &: \text{probability of exactly one } s_l \; i\text{-step mutations at locus } l \\
1 - \mu_l &: \text{probability of one non-mutation at locus } l \\
(1 - \mu_l)^{s_l} &: \text{probability of } s_l \text{ non-mutations at locus } l \\
H_0 &: \text{hypothesis that ancestor and offspring are related} \\
H_1 &: \text{hypothesis that ancestor and offspring are unrelated} \\
m &: \text{number of transmission events under } H_0 \text{ and } H_1 \\
\binom{m}{s_l} &: \text{binomial coefficient `} m \text{ choose } s_l\text{'} \\
LR &= \frac{P(E|H_0)}{P(E|H_1)}
\end{aligned}
$$

$$
LR_{\text{ only 1-steps}} = \frac{\prod_{l \in L} \left[\binom{m}{s_l} \left((\frac{\mu_l}{2})^{s_l} (1 - \mu_l)^{m - s_l} \right) \right]}{f(\text{offspring})} \tag{2}
$$

Example

$$\begin{aligned}
\text{ancestor haplotype} &: \{L_1 : \text{“10”}, L_2 : \text{“11”}\} \\
\text{offspring haplotype} &: \{L_1 : \text{“10”}, L_2 : \text{“12”}\} \\
f(\text{offspring}) &: 1/100;\ \text{frequency of offspring's haplotype} \\
m &: 1 \\
\mu &: \{L_1 : \{1 : 0.1, 2 : 0.05, 3 : 0.01\}, L_2 : \{1 : 0.1, 2 : 0.05, 3 : 0.01\}\} \\
(1-\mu) &: \{L_1 : 0.84, L_2 : 0.84\} \\
L &: \{L_1, L_2\} \\
s &: \{L_1 : 0, L_2 : 1\}
\end{aligned}$$

$$P(E|H_0) = \prod_{l \in L} \sum_{S_{l,i} \in p(s_l)} \left[\binom{m}{rank(S_{l,i})} \left(\prod_{s_{l,i,j} \in S_{l,i}} \left(\frac{\mu_{l,s_{l,i,j}}}{2} \right) \times (1-\mu_l)^{m-rank(S_{l,i})} \right) \right] \quad \Bigg| \; L = \{L_1, L_2\}$$

$$P(E|H_0) = \sum_{S_{L_1,i} \in p(s_{L_1})}^{L_1} [\dots] \times \sum_{S_{L_2,i} \in p(s_{L_2})}^{L_2} [\dots] \quad \Bigg| \; p(s_{L_1}) = \{\emptyset\} \text{ and } p(s_{L_2}) = \{\{1\}\}$$

$$\sum_{S_{L_1,i} \in p(s_{L_1})}^{L_1} [\dots] = \left[\binom{m}{rank(\emptyset)} \left(\prod_{s_{L_1,0,j} \in \emptyset} \left(\frac{\mu_{L_1,s_{L_1,0,j}}}{2} \right) \times (1-\mu_{L_1})^{m-rank(\emptyset)} \right) \right]_{S_{L_1,0}=\emptyset}$$

$$\sum_{S_{L_1,i} \in p(s_{L_1})}^{L_1} [\dots] = \left[\binom{1}{0} \left(1 \times (1-\mu_{L_1})^1 \right) \right]_{S_{L_1,0}=\emptyset} = 1 - \mu_{L_1}$$

$$\sum_{S_{L_2,i} \in p(s_{L_2})}^{L_2} [\dots] = \left[\binom{m}{rank(\{1\})} \left(\prod_{s_{L_2,0,j} \in \{1\}} \left(\frac{\mu_{L_2,s_{L_2,0,j}}}{2} \right) \times (1-\mu_{L_2})^{m-rank(\{1\})} \right) \right]_{S_{L_2,0}=\{1\}}$$

$$\prod_{s_{L_2,0,j} \in \{1\}} \left(\frac{\mu_{L_2,s_{L_2,0,j}}}{2} \right) = \left(\frac{\mu_{L_2,s_{L_2,0,0}}}{2} \right)_{s_{L_2,0,0}=1} = \left(\frac{\mu_{L_2,1}}{2} \right)$$

$$\sum_{S_{L_2,i} \in p(s_{L_2})}^{L_2} [\dots] = \left[\binom{1}{1} \left(\left(\frac{\mu_{L_2,1}}{2} \right) \times (1-\mu_{L_1})^0 \right) \right] = \frac{\mu_{L_2,1}}{2}$$

$$P(E|H_0) = \sum_{S_{L_1,i} \in p(s_{L_1})}^{L_1} [\dots] \times \sum_{S_{L_2,i} \in p(s_{L_2})}^{L_2} [\dots] = (1-\mu_{L_1}) \times \frac{\mu_{L_2,1}}{2} = 0.84 \times 0.05 = 0.042$$

$$P(E|H_1) = f(\text{offspring}) = 0.01$$

$$LR = \frac{P(E|H_0)}{P(E|H_1)} = \frac{0.042}{0.01} = 4.2$$